Enrollment No: ______________________**Exam Seat No:** _____________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Differential and Integral Calculus

Subject Code: 4SC04DIC1 **Branch: B.Sc. (Mathematics)**

Semester: 4 **Date: 04/05/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) Define: Gradient (02)

b) If f is solenoidal vector then what is the value of $grad(divf)$=______. (01)

c) True/False: Curl is a vector quantity. (01)

d) Evaluate: $\int_0^2\int_0^2 (x+y)\,dy\,dx$ (02)

e) Evaluate: $\int_1^2\int_0^1\int_2^3 xy^2z^3\,dz\,dy\,dx$ (02)

f) State Stoke's theorem. (02)

g) Define: Curvature (02)

h) Form the partial differential equation by eliminating the arbitrary constants from $z=(x+a)(y+b)$. (02)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2$ at the point $P(1,1,1)$ in the direction of line $\frac{x-1}{2} = \frac{y-3}{-2} = z$. (05)

b) For which value of component v_3 is $v = e^x cosy\, i + e^x siny\, j + v_3\, k$ solenoidal. (05)

c) Find $\nabla\phi$, where $\phi = \log(x^2 + y^2 + z^2)$ at (5,1,2). (04)

Q-3 Attempt all questions (14)

a) Define: Irrotational and find constants a, b, c if $\vec{v} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational function. (05)

b) Find the unit outward drawn normal to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at point $(3,1,-4)$. (05)

c) Find the equation of tangent plane and normal line at point $(-2,1,-3)$ to the ellipsoid $\frac{x^2}{4}+y^2+\frac{z^2}{9}=3$. (04)

Q-4 Attempt all questions **(14)**

a) If u, v are vector point functions and ϕ is a scalar point function then prove that $div(u\times v)=(curl u)\cdot v-u\cdot(curl v)$. (07)

b) If $\vec{v}=xi+yj+zk$ then show that curl$\vec{v}=\bar{0}$. (05)

c) State Green's theorem. (02)

Q-5 Attempt all questions **(14)**

a) Verify Green's theorem for $\oint_C\left[\left(x^2-2xy\right)dx+\left(x^2y+3\right)dy\right]$, where C is the boundary of the region bounded by the parabola $y=x^2$ and the line $y=x$ (08)

b) Evaluate $\iint_R (x+y)\,dA$, where R is Trapezoidal region with vertices $(0,0),(5,0)$, $\left(\frac{5}{2},\frac{5}{2}\right),\left(\frac{5}{2},-\frac{5}{2}\right)$ by using transformation $x=2u+3v$ and $y=2u-3v$. (06)

Q-6 Attempt all questions **(14)**

a) Evaluate $\iint_R 7xy^2\,dA$, where R is region bounded by $1\le x\le 2,\ 2\le y\le 3$. (05)

b) Evaluate $\int_0^2\int_{y^2}^4\left(x^2+y^2\right)dx\,dy$ by change the order of integration. (05)

c) Find the work done when a force $\bar{F}=3xy\,\hat{i}-y^2\,\hat{j}$ moves a particle from origin to $(1,2)$ along a parabola $y=2x^2$. (04)

Q-7 Attempt all questions **(14)**

a) Evaluate $\iiint_V dV$ where V is solid region bounded by $1\le x\le 2,\ 2\le y\le 4,\ 2\le z\le 5$ (05)

b) Evaluate $\iint_A x^2\,dx\,dy$ where A is the region in the first quadrant bounded by the hyperbola $xy=16$ and the lines $x=y, y=0, x=8$. (05)

c) Find the equation of tangent plane and normal line at point $(3,4,5)$ to the surface $x^2+y^2-4z=5$. (04)

Q-8 Attempt all questions **(14)**

a) Solve $\frac{\partial^2 z}{\partial x\partial y}=\sin x\cos y$, given that $\frac{\partial z}{\partial y}=-2\cos y$ when $x=0$ and $z=0$ when y is a multiple of π. (05)

b) Show that the radius of curvature at any point on the cardioids $r=a(1-\cos\theta)$ is (05)

$\frac{2}{3}\sqrt{2ar}$.

c) Form the partial differential equation by eliminating the arbitrary function from $z = xy + f\left(x^2 + y^2\right)$. (04)